GEOMETRY BY ANALYSIS

H. O. BARNES
Head of Department of Mathematics, Springfield High School,
Springfield, Illinois

As an observer of geometry work in high school and as a teacher of the subject, I became convinced a few years ago that the method commonly used fails to induce the pupils to study constructively, and that this method also fails to lead pupils to organize and express the results of this study clearly.

I noticed the large number of pupils in every geometry class who fail to get the real significance of the argument, and therefore really get nothing from the study. I also noticed a smaller number who master the set demonstration so easily that they are idle while the majority of the class are struggling with the problem. I noticed also the large group coming between these two extremes who master the work by dint of application to the set demonstration in which the argument is gone over repeatedly until fairly fixed.

I believe this is a fair statement of a situation that exists in most of the geometry classes in our high schools. It is a situation that was particularly depressing to me in my own classes, and I had the feeling that, while many of my pupils could demonstrate the theorems discussed in the classroom, few of them could apply this same procedure to new work of like grade. I think this feeling is common to teachers of the subject.

As I see it, geometry instruction should lead pupils to develop, each for himself, a large part of the demonstration, taking into account individual differences, in order to bring

to the pupils the power to think logically, to express this thinking clearly, and to apply this same kind of thinking to other work.

It occurred to me that the primary cause of most of the trouble in the study of geometry comes from the lack of harmony in the method of development and in the method of expression. Practically all of the formal demonstration in geometry in our schools is synthetic, while the preparation for it is analytic by nature.

I attacked the problem of discovering some means by which pupils could express their geometrical thought without the constant "backing-up" process involved in the common synthetic demonstration.

At first the pupils wrote out the analysis for the demonstration in paragraph form, merely as a preparation for the synthetic demonstration. I modified the plan of instruction from time to time as it became clearer to me that good geometrical thinking and expression are interdependent, and I finally devised the form paper shown on page 617 for the formal demonstration by analysis.

In connection with this form of expression I also devised a plan of suggesting, where necessary, the data from which pupils build up the demonstration so given, as to secure complete harmony of the expression and the preparation.

This plan of instruction depends upon a certain general preparation for it which at first may seem onerous and doubtful; but it is very important. I make both the development and the expression of a demonstration analytic. There is no attempt to adhere to the idea of a formal syllogism, but the pupils as a preparation for all demonstration are taught that every step must name first the conclusion, which we call a Major. (This, of course, is a misnomer so far as a syllogism is concerned, but is better than "conclusion," since it changes from step to step.) Then the pupils are taught that each

Major depends upon one or several minor premises, which we call Minors. The pupils are taught also that there must be an authority for the truth of the Major depending upon the establishment of the truth of the Minors. And, finally, the pupils are taught that every Minor must have a direct authority, which settles the argument, or else each Minor in turn must become a Major in the next step. (Refer to demonstration, page 617.)

Let us now take a particular theorem and follow the procedure in the classroom. First the teacher makes the assignment, which may be from the book, or may be written on the board. Suppose this assignment is the demonstration for the theorem: "If the opposite sides of a quadrilateral are equal, the figure is a parallelogram." The teacher places the following study outline on the board:

OUTLINE:

Theorem: (Left to the student.)

Given: $DC = AB$ (Figure here.)

$AD = BC$

Required:

$AB \parallel DC$

$AD \parallel BC$

Demonstration:

(Since the required contains two things exactly alike, prove one of them, then state that the other is proved by the same kind of argument.)

First Major depends upon one Minor involving angles; its authority is 73.

Second Major depends upon one Minor; its authority is 37.

Third Major depends upon three Minors whose authorities are 26, 26, and 34.

[These numbers refer to paragraphs in the text.]

You will notice that in the first suggestion the pupil is required to make the application of a cited authority in the expression of the Minor upon which the Major depends. This is true also in the second suggestion, but the kind of thing to be found and applied is different, since the first Major depends

upon equality of angles and the second Major depends upon the congruency of triangles. The third suggestion is such that the pupil is required to seek the three Minors upon which the Major depends and also the authority for their application, knowing as a basis for this search the authorities for the Minors.

I find it advisable to make the suggestions in such a way that the pupil must seek different kinds of things in the different parts of the demonstration. The teacher, knowing his class, must suit the suggestions to the class needs. I make the suggestions on the board a minimum and amplify personally where needed. The following rules guide:

1. The suggestions should be such that, together with the quantity of work, they will keep the brightest pupils strenuously employed.

2. The dullest pupils should be able to make some definite progress.

3. The suggestions should leave as much as possible to the pupils.

4. At the same time the suggestions should make it possible for all or nearly all the pupils to work up the demonstration.

In my classes the time divisions for work are not iron-clad, some propositions requiring more time for preparation, more explanatory matter, and more time for expression, than others. The following is perhaps the best approximation of time divisions: The first ten minutes of the sixty-minute period are given to recitation—a discussion of previous work, a summary, an explanation, etc. The next twenty minutes are devoted to writing on form paper the analytical demonstration of the proposition studied the day before. In the last thirty minutes the pupils prepare the work for the next day. The unit of the work begins with this thirty minutes.

The pupil begins his study by ruling up on scratch paper a form like that on which he is going to write his work the next day, and fills in "Theorem," "Given," "Required," etc. The suggestions sometimes leave all these to the pupil.

The pupil now determines his first Major, which he obtains from the "Required." By looking up the reference numbers for the authority, he finds he must fix two particular angles and uses them in expressing the Minor upon which the Major depends. As soon as this matter is settled, he discovers he has no direct authority for this Minor. He knows by the plan of procedure this Minor now becomes the Major in the next step. He proceeds with the second Major in the same way, the third, and so forward. In this way the pupil has a very definite problem throughout the course of the demonstration. He knows that his argument is settled at the exact point where every Minor has a direct authority. The next day he writes the complete demonstration on form paper. On the following page is such a paper taken from current work:

This work in geometry is in three classes, a total of seventy-five pupils. Among these are perhaps four or five "professional loafers," two or three so-called "subnormals," and four or five very bright children.

In contrast with other classes which I have had in geometry, it is my conviction that there is not a single individual in these classes who does not come to the classroom anticipating a delightful hour. There is not one who does not take the initiative in his own work. And when I consider attendance records, natural aptitude, sickness handicaps, etc., I believe there is not an individual who will not accomplish all that could reasonably be expected of him this semester. The whole experiment at the present writing has resulted in delightful working relations between pupils and teacher, both from an aesthetic viewpoint and from that of effectiveness of thinking on the part of the pupils.

This work provides for the brightest pupils in the class, and at the same time the dullest pupils make some definite progress. The possibility of meeting this problem of individual differences comes from the fact that the procedure in original work

NAME Wilbur Hahn Class 10' Period 1 Date 1-20-19 Mark

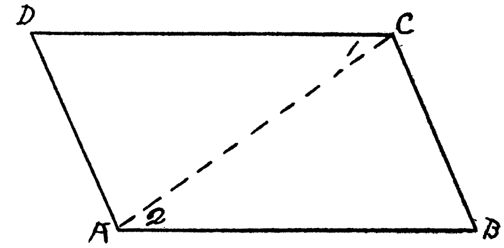
GEOMETRY DEMONSTRATION BY ANALYSIS

Theorem If the opposite sides of a quadrilateral are equal the figure is a parallelogram.

Given AB = DC
AD = BC

Required ABCD is a ▭
or AB ∥ DC
AD ∥ BC

Figure



Argument: Major	Argument: Minor	Authority: For Minor	Authority: For Major
	Draw the diagonal AC		
1. AB ∥ DC	If ∠1 = ∠2		If two lines are cut by a transversal so the alternate interior angles are equal the lines are parallel.
2. ∠1 = ∠2	If △ABC ≅ △ACD		corresponding parts
3. △ABC ≅ △ACD	If AB = DC AD = BC AC = AC	Given Given Identical	Two triangles are congruent if the three sides of the one equal respectively the three sides of the other.
∴ AB ∥ DC and AD ∥ BC	by the same argument		

is exactly the same as that of the "sequential theorems," and the bright pupils readily apply this working plan to additional work given them.

The development of this plan of instruction has been greatly facilitated by working in a school where supervised study is in vogue. I have not measured in a statistical way the results of this study by analysis as compared with the usual synthetic work, but I am sure of several things from observation:

1. Pupils become skilful in discerning the new problem constantly arising when Minors become Majors.

2. Pupils like to demonstrate when they feel they are doing a large part of the work constructively themselves.

3. Pupils have more initiative and are more independent, since the procedure keeps a very definite problem before them and they are not merely acquiescing in statements of a text.

4. Pupils are far more efficient in demonstration of originals, for the simple reason they are familiar with the true method of attack.

5. Pupils carry this analysis into other subjects, making it possible for them to grasp the problem and seek the premises for its solution.